Multichannel Sampling of Signals with Finite Rate of Innovation

Hojjat Akhondi Asl, Pier Luigi Dragotti and Loic Baboulaz

I. EXTENDED ABSTRACT

In this paper we present a possible extension of the theory of sampling signals with finite rate of innovation (FRI) to the case of multichannel acquisition systems. Most of the papers on sparse sampling [3], [2], [6], [4] focus on a single-channel acquisition model. However, modern and fast Analogue-to-Digital Converters (ADC) use interleaved multichannel converters. This allows a reduction in the complexity of the devices while keeping higher rates of conversion. Given the practical importance of multichannel acquisition devices, it is natural to investigate extensions of sparse sampling theories to the multichannel scenario. The critical issue in multichannel sampling (see Figure 1) is the precise synchronization of the various channels since different devices introduce different drifts and different gains (due to imperfections of electronic circuits for example) that need to be estimated together with the signal itself. We pose both the synchronization stage and the signal reconstruction stage as a parametric estimation problem and demonstrate that a simultaneous exact synchronization of the channels and reconstruction of the FRI signal is possible.

In this paper, we will focus on a specific class of kernels, used in [4], that are able to reproduce real or complex exponentials. Our goal is to have a reconstruction system that can perfectly retrieve both the input signal and the unknown delays and gains. By setting two parameters to be common between the exponents of the ith channel, with $i = 2, 3, \ldots, M$, with respect to the reference channel, the unknown gain and the delay factor can be calculated. This reveals that, independently of $x(t)$, it is possible to synchronize the two channels exactly from the samples $y_{i,n}$. A multichannel acquisition system achieves perfect reconstruction of FRI signals with a sampling rate proportional to $1/TM$. Thus, perfect reconstruction is achieved at lower sampling rates.

We also show in our paper that a multichannel system (two and three channels in our case) is more resilient to noise than a single channel one. Since for both single and multichannel set-ups, we have a standard parametric estimation problem, we use Cramér-Rao bounds (CRB) to compare the minimum bounds of the different set-ups. As the large number of unknown parameters leads to a fairly large Fisher information matrices, it is simpler to evaluate the CRB numerically for all cases. The CRB for the estimation of the a FRI signal consisting of 3 Diracs with known amplitudes for $M = 1, 2$ and 3, with a fixed number of samples $N = 20$, are shown in Figure 2(a). Interestingly, the results reveal that the CRB improves with the number of channels. More precisely, the CRB improvement when going from single-channel to two channels is approximately 0.86dB, while the improvement when going from single channel to three channel system is approximately 1.1dB. It is interesting to see that, despite the fact that the unknown delays need to be estimated in order to synchronize the channels, there is still a noticeable gain by using multichannel sampling set-up when compared to the single channel sampling set-up. Furthermore, in Figure 2(b) we show

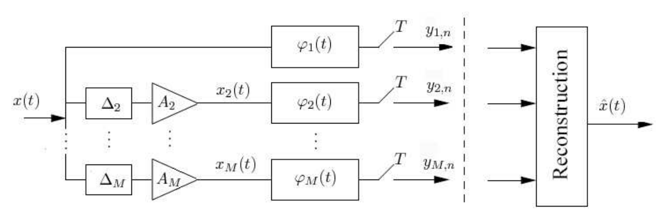
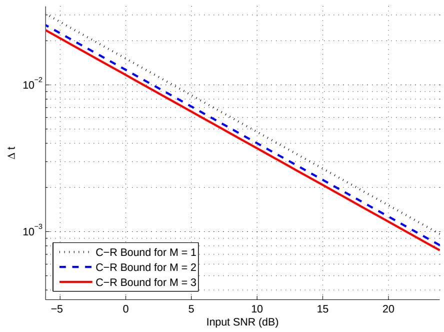
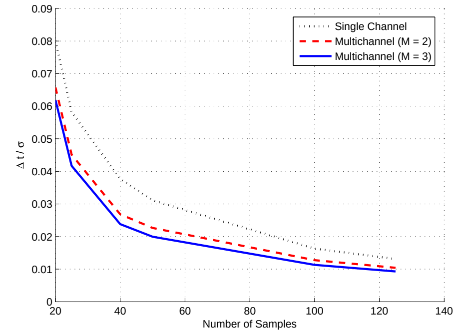


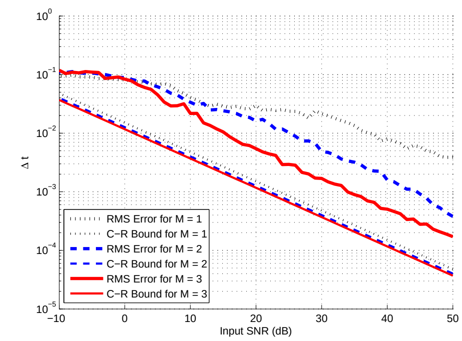
Fig. 1. Multichannel sampling set-up. Here, the continuous-time signal $x(t)$ is received by multiple channels with multiple acquisition devices. The samples $y_{i,n}$ from each channel are utilized jointly for the reconstruction process.



(a)



(b)



(c)

Fig. 2. (a) CRB for single channel and multichannel sampling systems. The input SNR is calculated as $10log_{10}\frac{||y||^2}{\sigma^2}$ where σ^2 is the noise variance and Δt is the uncertainty on the estimated locations. (b) Theoretical uncertainties on the estimated locations with varying sampling rates. (c) Numerical results with single and multichannel sampling. Dirac locations are set at 0.5, 0.6 and 0.7 set for all cases. For the sake of simplicity, the introduced channel gains are all set to be equal and known a-priori. The delays Δ_2 and Δ_3 are fixed at $\frac{T}{2}$ and T respectively.

the CRB of each sampling system at varying sampling rates. We can see that at a given uncertainty of the estimated locations, there is a reduction in the number of samples needed when going from single-channel to multichannel sampling systems. For example, at the reconstruction quality of $\frac{\Delta t}{\sigma} = 0.04$, the number of samples could be reduced from 38 samples to 27 samples when going from the single channel to the three channel set-up.

To analyze the performance of the reconstruction algorithm, Figure 2(c) presents some actual numerical results on the uncertainty of the estimated locations which are also compared against the theoretical bounds from Figure 2(a). The locations of the Diracs are obtained using a variation of the annihilating filter method, known as the matrix-pencil method [5] and also the Cadzow's algorithm [1] to further denoise the surrogate measurements s_m. While none of the algorithms achieve the CRB, the obtained results show that the gain in performance with multichannel sampling over single channel sampling can be significant. For instance, at input SNR$= 15$dB, the gain in performance from single channel to three channels is approximately 4.4dB.

References

[1] T. Blu, P.L. Dragotti, M. Vetterli, Marziliano P., and Coulot L. Sparse sampling of signal innovations: Theory, algorithms and performance bounds. *IEEE Signal Processing Magazine*, 25(2):31–40, March 2008.

[2] E. Candès, J. Romberg, and T. Tao. Robust uncertainty principle: Exact signal reconstruction from highly incomplete frequency information. *IEEE Trans. Information Theory*, 52(2):489–509, February 2006.

[3] D.L. Donoho. Compressed sensing. *IEEE Trans. Information Theory*, 52(4):1289–1306, April 2006.

[4] P.L. Dragotti, M. Vetterli, and T. Blu. Sampling moments and reconstructing signals of finite rate of innovation: Shannon meets Strang-Fix. *IEEE Trans. on Signal Processing*, 55(5):1741–1757, May 2007.

[5] P. Stoica and R. Moses. *Spectral Analysis of Signals*. Prentice Hall, April 2005.

[6] M. Vetterli, P. Marziliano, and T. Blu. Sampling signals with finite rate of innovation. *IEEE Trans. Signal Processing*, 50(6):1417–1428, June 2002.